NDU

MAT 224
Calculus IV

Final Exam

Thursday, June 16, 2016
Duration: 2 hours

Name ____________________

Section ___________________

Instructor ________________

Cell phones are forbidden
You have 6 exercises and 8 pages.

1) (55 points) For each of the multiple choice questions below, circle the **letter** of the correct answer. If more than one letter is circled in the same problem, you will receive no credit for that problem.

Question A

Let $f(x,y) = \ln\left(e + \frac{xy^3 - x^4}{x^2 + y^2}\right)$. Then:

a) $f(x,y)$ can be extended to become continuous at (0,0) by defining $f(0,0) = 0$.
b) $f(x,y)$ can be extended to become continuous at (0,0) by defining $f(0,0) = e$.
c) $f(x,y)$ can be extended to become continuous at (0,0) by defining $f(0,0) = 1$.
d) $f(x,y)$ cannot be extended in any way to make it continuous at (0,0).

Question B

Let $w = f(x,y,z)$ be a differentiable function with $x = \sin u + \ln v$, $y = \sin v + \ln u$, $z = \frac{u}{v}$, then $\frac{\partial w}{\partial v}$ at $(u,v) = (\frac{\pi}{4}, \frac{\pi}{4})$ is :

a) $\frac{4}{\pi} f_x + \frac{\sqrt{2}}{2} f_y - \frac{4}{\pi} f_z$
b) $\frac{4}{\pi} f_x - \frac{\sqrt{2}}{2} f_y - \frac{4}{\pi} f_z$
c) $\frac{\sqrt{2}}{2} f_x + \frac{4}{\pi} f_y + \frac{4}{\pi} f_z$
d) $\frac{\sqrt{2}}{2} f_x - \frac{4}{\pi} f_y + \frac{4}{\pi} f_z$

Question C

The absolute minimum m and absolute maximum M of $f(x,y) = 2x^2 + y^2 - 4x$ on the disk $x^2 + y^2 \le 9$ have the values

a) $m = -2$; $M = 5$
b) $m = -2$; $M = 30$
c) $m = 5$; $M = 30$
d) $m = 5$; $M = 6$

Question D

Consider the following two surfaces S_1 and S_2 defined by $S_1 : -2x^2 - z^2 = 2y - 4$ and $S_2 : y^2 - x^2 = -z^2$. The equation of the line tangent to the curve of intersection of the two surfaces at the point $(\frac{2}{\sqrt{3}}, 0, \frac{2}{\sqrt{3}})$ is given by

a) $x = -\frac{8}{\sqrt{3}}t + \frac{2}{\sqrt{3}}, y = 16t, z = -\frac{8}{\sqrt{3}}t + \frac{2}{\sqrt{3}}$, t is a real parameter

b) $x = -\frac{8}{\sqrt{3}}t + \frac{2}{\sqrt{3}}, y = -16t, z = -\frac{8}{\sqrt{3}}t + \frac{2}{\sqrt{3}}$, t is a real parameter

c) $x = \frac{8}{\sqrt{3}}t + \frac{2}{\sqrt{3}}, y = 16t, z = -\frac{8}{\sqrt{3}}t + \frac{2}{\sqrt{3}}$, t is a real parameter

d) $x = \frac{8}{\sqrt{3}}t + \frac{2}{\sqrt{3}}, y = -16t, z = \frac{8}{\sqrt{3}}t + \frac{2}{\sqrt{3}}$, t is a real parameter

Question E

Evaluate $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} e^{x^2+y^2}\,dxdy$

a) $\pi(e^9 - 1)$

b) $\pi(e^9 + 1)$

c) $\frac{\pi}{2}(e^9 - 1)$

d) $\frac{\pi}{2}(e^9 + 1)$

Question F

Evaluate $\int_0^1 \int_{-2}^0 \int_{-x}^2 \frac{\cos(y^2)}{1+z^2}\,dydxdz$

a) $-\frac{\pi}{4}\sin 4$

b) $\frac{\pi}{4}\sin 4$

c) $-\frac{\pi}{8}\sin 4$

d) $\frac{\pi}{8}\sin 4$

Question G

Consider the solid S bounded laterally by $x^2 + y^2 = 1$, below by the xy-plane and above by $z^2 = 4x^2 + 4y^2$. Then $\iiint_S x^2 \, dV$ in cylindrical coordinates is given by

a) $\int_0^{2\pi}\int_0^2\int_0^{2r} r^3 \cos^2(\theta) dz\, dr\, d\theta$

b) $\int_0^{2\pi}\int_0^1\int_0^{4r^2} r^3 \cos^2(\theta) dz\, dr\, d\theta$

c) $\int_0^{2\pi}\int_0^1\int_0^{2r} r^3 \cos^2(\theta) dz\, dr\, d\theta$

d) $\int_0^{2\pi}\int_0^2\int_0^{4r^2} r^3 \cos^2(\theta) dz\, dr\, d\theta$

Question H

Find the work done by the field $\vec{F} = y\vec{i} + x\vec{j} + 3z\vec{k}$ along the path $(C_1) \cup (C_2)$ from (0, 1, 0) to (1, 2, 1) where (C_1) and (C_2) are the following curves:

(C_1): The curve $y = x^2 + 1$ in the xy-plane, $0 \le x \le 1$

(C_2): The line segment from (1, 2, 0) to (1, 2, 1)

a) $\frac{7}{2}$

b) $\frac{3}{2}$

c) 2

d) $\frac{5}{6}$

Question I

Apply Green's theorem to evaluate $\int_C (x^2 - y)dx + (2xy + y^2)dy$, where C is the boundary of the region in the first quadrant enclosed by the x-axis, the line $x = 1$, and the curve $y = x^2$.

a) $-\frac{8}{15}$

b) $\frac{6}{5}$

c) $-\frac{2}{15}$

d) $\frac{8}{15}$

Question J Consider the integral $\iint_R (x+y)\cos(3y-x)dydx$, where R is the triangular region in the xy-plane bounded by the lines $y = 0, y = -x,$ and $3y - x = 4$. Let G be the region in the uv-plane which is the image of R under the transformation $u = 3y - x$ and $v = x + y$. Then

Part 1 $\iint_R (x+y)\cos(3y-x)dydx =$

a) $-\iint_G 4v\cos u\,dudv$

b) $\iint_G \frac{1}{4}v\cos u\,dudv$

c) $-\iint_G \frac{1}{4}v\cos u\,dudv$

d) $\iint_G 4v\cos u\,dudv$

Part 2: The region G in the uv-plane is bounded by the lines:

a) $u = 4,\ u = -v,\ v = 0$

b) $3v - u = 4,\ u = v,\ v = 0$

c) $u = 4,\ u = v,\ v = 3$

d) $u = 3,\ u = 3v,\ v = 0$

2) (24 points)

a) (12points) Let D be the region that is bounded from below by $z = 1$ and from above by $x^2 + y^2 + z^2 = 4$. Sketch D and **set up** triple integrals in spherical coordinates representing its volume according to the order of integration $d\rho\, d\phi\, d\theta$.

b) (12 points) Set up triple integrals in spherical coordinates representing its volume according to the order of integration $d\phi\, d\rho\, d\theta$

3) (8 points) The derivative of a function $f(x, y, z)$ at a point P is greatest in the direction of $\vec{v} = \vec{i} - 2\vec{j} + 2\vec{k}$. In this direction, the value of the derivative is 2. Find $\vec{\nabla} f$ at P.

4) (5 points) Find the line integral of the function $f(x, y, z) = xy + y + z$ over the path $\vec{r}(t) = 2t\vec{i} + t\vec{j} + (2 - 2t)\vec{k},\ \ 0 \le t \le 1$

5)(10 points) Consider the field

$$\vec{F}(x,y,z) = (2x\cos y - 2z^3)\vec{i} + (3 + 2ye^z - x^2 \sin y)\vec{j} + (y^2 e^z - 6xz^2)\vec{k}$$

a) (7 points) Show that $\vec{F}$ is conservative and find a potential function for $\vec{F}$

b) (3 points) Evaluate the work done by $\vec{F}$ along any smooth path from $A(0,0,0)$ to $B(1,\pi,0)$

6) (8 points) Use Green's theorem to find the outward flux for the field $\vec{F} = 2xy^2\vec{i} + x^2y^2\vec{j}$ across the curve (C): the boundary of the triangular region bounded by the lines $y = x$, $y = -x$ and $y = 1$